Estimation of the failure probability of a thermal-hydraulic passive system by means of Artificial Neural Networks and quadratic Response Surfaces

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ABSTRACT: In this paper, Artificial Neural Network (ANN) and quadratic Response Surface (RS) empirical regression models are used as fast-running surrogates of a thermal-hydraulic (T-H) system code to reduce the computational burden associated with the estimation of the functional failure probability of a T-H passive system. The ANN and quadratic RS models are constructed on a limited-size set of input/output data examples of the nonlinear relationships underlying the original T-H code; once built, these models are used for performing, in an acceptable computational time, the numerous system response calculations needed for an accurate uncertainty propagation and failure probability estimation. An application to the functional failure analysis of an emergency passive decay heat removal system in a simple steady-state model of a Gas-cooled Fast Reactor (GFR) is presented.

1 INTRODUCTION

Modern nuclear reactor concepts make use of passive safety features (Fong et al. 2009), which do not need external input (especially energy) to operate (IAEA 1991) and, thus, are expected to improve the safety of nuclear power plants because of simplicity and reduction of both human interactions and hardware failures (Nayak et al. 2009).

However, the uncertainties involved in the modelling and functioning of passive systems are usually larger than for active systems. This is due to: i) the random nature of several of the physical phenomena involved in the functioning of the system (aleatory uncertainty); ii) the incomplete knowledge on the physics of some of these phenomena (epistemic uncertainty) (Apostolakis 1990).

Due to these uncertainties, the physical phenomena involved in the passive system functioning (e.g., natural circulation) might develop in such a way to lead the system to fail its function: actually, deviations in the natural forces and in the conditions of the underlying physical principles from the expected ones can impair the function of the system itself (Burgazzi 2007).

In this view, a passive system fails to perform its function when deviations from its expected behavior lead the load imposed on the system to exceed its capacity (Burgazzi 2007). In the reliability analysis of such functional failure behavior, the passive system is modeled by a detailed, mechanistic T-H system code and the probability of failing to perform the required function is estimated based on a Monte Carlo (MC) sample of code runs which propagate the *epistemic* (state-of-knowledge) uncertainties in the model and numerical values of its parameters/variables (Mackay et al. 2008, Patalano et al. 2008, Arul et al. 2009, Mathews et al. 2009, Fong et al. 2009).

Since the probabilities of functional failure of passive systems are generally very small (e.g., of the order of 10^{-4}), a large number of samples is necessary for acceptable estimation accuracy (Schueller 2007); given that the time required for each run of the detailed, mechanistic T-H system code is of the order of several hours (Fong et al. 2009), the MC simulation-based procedure typically requires considerable computational efforts.

A viable approach to overcome the computational burden associated to the analysis is that of resorting to fast-running, surrogate regression models, also called response surfaces or meta-models, to approximate the input/output function implemented in the long-running T-H model code, and then substitute it in the passive system functional failure analysis. The construction of such regression models entails running the T-H model code a predetermined, reduced number of times (e.g., 50-100) for specified values of the uncertain input parameters/variables and collecting the corresponding values of the output of interest; then, statistical techniques are employed for fitting the response surface of the regression model to the input/output data generated in the previous step. Several examples can be found in the open literature concerning the application of surrogate meta-models in reliability problems. In (Liel et al. 2009), polynomial Response Surfaces (RSs) are employed to evaluate the failure probability of structural systems; in (Arul et al. 2009, Fong et al. 2009, Mathews et al. 2009), *linear* and *quadratic* polynomial RSs are employed for performing the reliability analysis of T-H passive systems in advanced nuclear reactors; in (Cardoso et al. 2008), Artificial Neural Networks (ANNs) are trained to provide *local* approximations of the failure domain in structural reliability problems; in (Marrel et al. 2009, Storlie et al. 2009), various regression models (including Gaussian meta-models) are built to calculate global sensitivity indices for a complex hydrogeological model simulating radionuclide transport in groundwater.

In this work, the possibility of using Artificial Neural Networks (ANNs) and quadratic Response Surfaces (RSs) to reduce the computational burden associated to the functional failure analysis of a natural convection-based decay heat removal system of a Gas-cooled Fast Reactor (GFR) (Pagani et al. 2005) is investigated. To keep the practical applicability in sight, a small set of input/output data examples is considered available for constructing the ANN and quadratic RS models: different sizes of the (small) data sets are considered to show the effects of this relevant practical aspect. The comparison of the potentials of the two regression techniques in the case at hand is made with respect to the estimation of i) the Probability Density Function (PDF) of the temperature of the naturally circulating coolant in the passive system, ii) the 95th percentile of the naturally circulating coolant temperature and iii) the functional failure probability of the passive system.

The paper organization is as follows. In Section 2, the concepts of functional failure analysis for T-H passive systems are synthetically summarized. Section 3 briefly presents the problem of empirical regression modeling. In Section 4, the case study of literature concerning the passive cooling of a GFR is presented. In Section 5, the results of the application of ANNs and quadratic RSs to the functional failure analysis of the T-H passive system of Section 4 are reported. Conclusions are provided in the last section.

2 FUNCTIONAL FAILURE ANALYSIS OF T-H PASSIVE SYSTEMS

The basic quantitative steps of the functional failure analysis of a T-H passive system are (Bassi & Marquès 2008):

1 Detailed modeling of the passive system response by means of a deterministic, best-estimate (typically long-running) T-H code.

2 Identification of the parameters/variables, models and correlations (i.e., the inputs to the T-H code) which contribute to the uncertainty in the results (i.e., the outputs) of the best estimate T-H calculations.

3 Propagation of the uncertainties through the deterministic, long-running T-H code in order to estimate the functional failure probability of the passive system.

Step 3. above relies on multiple (e.g., many thousands) evaluations of the T-H code for different combinations of system inputs; this can render the associated computing cost prohibitive, when the running time for each T-H code simulation takes several hours (which is often the case for T-H passive systems).

The computational issue may be tackled by replacing the long-running, original T-H model code by a fast-running, surrogate regression model (properly built to approximate the output from the true system model). In this paper, classical three-layered feed-forward Artificial Neural Networks (ANNs) (Bishop 1995) and quadratic Response Surfaces (RSs) (Liel et al. 2009) are considered for this task.

3 RESPONSE SURFACES AND ARTIFICIAL NEURAL NETWORKS

Let us consider a generic meta-model to be built for performing the task of nonlinear regression, i.e., estimating the nonlinear relationship between a vector of input variables $\boldsymbol{x} = \{x_1, x_2, ..., x_j, ..., x_{n_i}\}$ and a vector of output targets $\boldsymbol{y} = \{y_1, y_2, ..., y_l, ..., y_{n_o}\}$, on the basis of a *finite* (and possibly *small*) set of input/output data examples (i.e., patterns), $D_{train} = \{(\boldsymbol{x}_p, \boldsymbol{y}_p), p = 1, 2, ..., N_{train}\}$ (Zio 2006). It can be assumed that the target vector $\boldsymbol{y}$ is related to the input vector $\boldsymbol{x}$ by an unknown nonlinear deterministic function $\boldsymbol{\mu}_y(\boldsymbol{x})$ corrupted by a noise vector $\boldsymbol{\varepsilon}(\boldsymbol{x})$, i.e.,

$$\boldsymbol{y}(\boldsymbol{x}) = \boldsymbol{\mu}_y(\boldsymbol{x}) + \boldsymbol{\varepsilon}(\boldsymbol{x}) \tag{1}$$

In the present case of T-H passive system functional failure probability assessment the vector $\boldsymbol{x}$ contains the relevant uncertain system parameters/variables, the nonlinear deterministic function $\boldsymbol{\mu}_y(\boldsymbol{x})$ represents the complex, long-running T-H mechanistic model code (e.g., RELAP5-3D), the vector $\boldsymbol{y}(\boldsymbol{x})$ contains the output variables of interest for the analysis and the noise $\boldsymbol{\varepsilon}(\boldsymbol{x})$ represents the errors introduced by the numerical methods employed to calculate $\boldsymbol{\mu}_y(\boldsymbol{x})$; for simplicity, in the following we assume $\boldsymbol{\varepsilon}(\boldsymbol{x}) = \boldsymbol{0}$ (Storlie et al. 2009).

The objective of the regression task is to estimate $\boldsymbol{\mu}_y(\boldsymbol{x})$ in (1) by means of a regression function $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{w}^*)$ depending on a set of parameters $\boldsymbol{w}^*$ to be properly determined on the basis of the available data set D_{train}; the algorithm used to calibrate the set of parameters $\boldsymbol{w}^*$ is obviously dependent on the nature of the regression model adopted, but in general it aims at minimizing the mean (absolute or quadratic) error between the output targets of the original T-H code, $\boldsymbol{y}_p = \boldsymbol{\mu}_y(\boldsymbol{x})$, $p = 1, 2, ..., N_{train}$, and the output vectors of the regression model, $\boldsymbol{y}_p = \boldsymbol{f}(\boldsymbol{x}_p, \boldsymbol{w}^*)$, $p = 1, 2, ..., N_{train}$.

Once built, the regression model $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{w}^*)$ can be used in place of the T-H code to calculate any quantity of interest Q, such as the 95th percentile of a physical variable critical for the system under analysis (e.g., the fuel cladding temperature) or the functional failure probability of the passive system.

In this work, the capabilities of quadratic Response Surface (RS) and three-layered feed-forward Artificial Neural Network (ANN) regression models are compared in the computational tasks involved in the functional failure analysis of a T-H passive system. In extreme synthesis, quadratic RSs are polynomials containing linear terms, squared terms and possibly two-factors interactions of the input variables (Liel et al. 2009); the RS adaptable parameters $\boldsymbol{w}^*$ are usually calibrated by straightforward least squares methods. ANNs are computing devices inspired by the function of the nerve cells in the brain (Bishop 1995). They are composed of many parallel computing units (called neurons or nodes) interconnected by weighed connections (called synapses). Each of these computing units performs a few simple operations and communicates the results to its neighbouring units. From a mathematical viewpoint, ANNs consist of a set of nonlinear (e.g., sigmoidal) basis functions with adaptable parameters $\boldsymbol{w}^*$ that are adjusted by a process of *training* (on many different input/output data examples), i.e., an iterative process of regression error minimization (Rumelhart et al. 1986). The particular type of ANN employed in this paper is the classical three-layered feed-forward ANN trained by the error back-propagation algorithm.

4 CASE STUDY

The case study considered in this work concerns the natural convection cooling in a Gas-cooled Fast Reactor (GFR) under a post-Loss Of Coolant Accident (LOCA) condition (Pagani et al. 2005). The reactor is a 600-MW GFR cooled by helium whose design has been the subject of study in the past several years at the Massachussets Institute of Technology (MIT) (Pagani et al. 2005).

A GFR decay heat removal configuration is shown schematically in Figure 1; in the case of a LOCA, the long-term heat removal is ensured by natural circulation in a given number N_{loops} of identical and parallel loops; only one of the N_{loops} loops is reported for clarity of the picture: the flow path of the cooling helium gas is indicated by the black arrows. The loop has been divided into $N_{sections}$ = 18 sections for numerical calculation; technical details about the geometrical and structural properties of these sections can be found in (Pagani et al. 2005).

In the present analysis, the average core power to be removed is assumed to be 18.7 MW, equivalent to about 3% of full reactor power (600 MW): to guarantee natural circulation cooling at this power level, a pressure of 1650 kPa in the loops is required in nominal conditions. Finally, the secondary side of the heat exchanger (i.e., item 12 in Figure 1) is assumed to have a nominal wall temperature of 90 °C (Pagani et al. 2005).

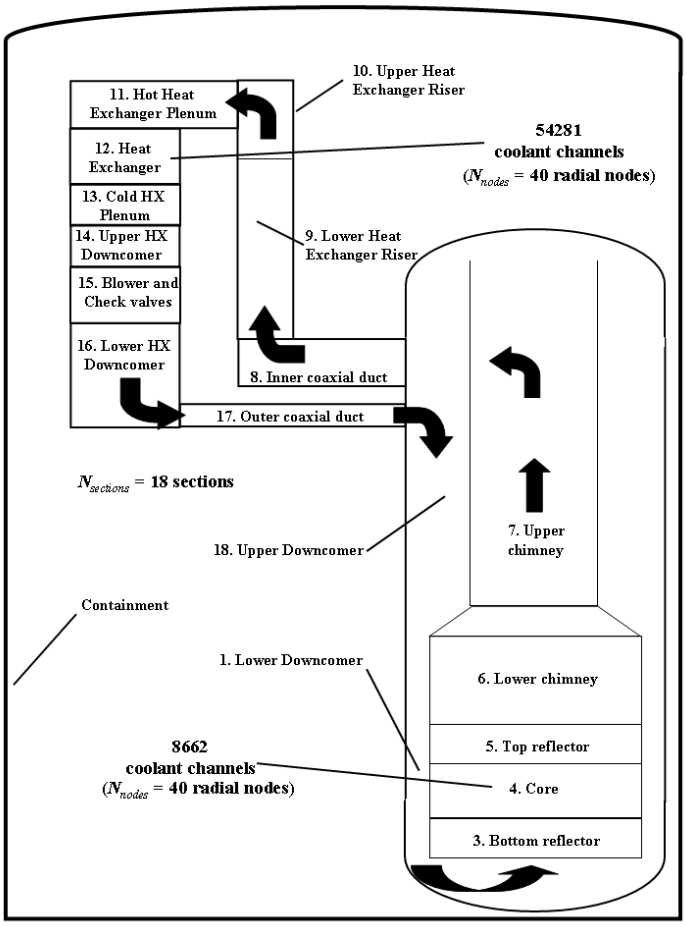

Figure 1. Schematic representation of one loop of the 600-MW GFR passive decay heat removal system (Pagani et al. 2005).

4.1 *Uncertainties*

Only epistemic uncertainties are considered in this work. Epistemic parameter uncertainties are associated to the reactor power level (x_1), the pressure in the loops after the LOCA (x_2) and the cooler wall temperature (x_3); epistemic model uncertainties are associated to the correlations used to calculate the Nusselt numbers (x_4, x_5 and x_6) and friction factors (x_7, x_8 and x_9) in the forced, mixed and free convection regimes, respectively. The consideration of these uncertainties leads to the definition of a vector $\boldsymbol{x}$ of nine uncertain inputs of the model $\boldsymbol{x} = \{x_j: j = 1, 2, ..., 9\}$, assumed described by normal distributions of known means and standard deviations (Table 1, Pagani et al. 2005).

Table 1. Epistemic uncertainties considered for the 600-MW GFR passive decay heat removal system of Figure 1 (Pagani et al. 2005).

	Variable	Mean, μ	Standard deviation (% of μ)
Parameter uncertainty	x_1	18.7 MW	1%
	x_2	1650 kPa	7.5%
	x_3	90 °C	5%
Model uncertainty	x_4	1	5%
	x_5	1	15%
	x_6	1	7.5%
	x_7	1	1%
	x_8	1	10%
	x_9	1	1.5%

4.2 *Failure criteria of the T-H passive system*

The passive decay heat removal system of Figure 1 is considered failed when the temperature of the coolant helium leaving the core (item 4 in Figure 1) exceeds either 1200 °C in the hot channel or 850 °C in the average channel: these values are expected to limit the fuel temperature to levels which prevent excessive release of fission gases and high thermal stresses in the cooler (item 12 in Figure 1) and in the stainless steel cross ducts connecting the reactor vessel and the cooler (items from 6 to 11 in Figure 1) (Pagani et al. 2005). Denoting by $T_{out,core}^{hot}(\boldsymbol{x})$ and $T_{out,core}^{avg}(\boldsymbol{x})$ the coolant outlet temperatures in the hot and average channels, respectively, the system failure event F can be written as follows:

$$F = \left\{\boldsymbol{x} : T_{out,core}^{hot}(\boldsymbol{x}) > 1200\right\} \cup \left\{\boldsymbol{x} : T_{out,core}^{avg}(\boldsymbol{x}) > 850\right\} \quad (2)$$

According to the notation of the preceding Section 3, $T_{out,core}^{hot}(\boldsymbol{x}) = y_1(\boldsymbol{x})$ and $T_{out,core}^{avg}(\boldsymbol{x}) = y_2(\boldsymbol{x})$ are the two target outputs of the T-H model.

5 RESULTS

In this Section, the results of the application of Artificial Neural Networks (ANNs) and quadratic Response Surfaces (RSs) for the estimation of the functional failure probability of the 600-MW GFR passive decay heat removal system in Figure 1 are illustrated. Some details about the construction of the ANN and quadratic RS regression models are given in Section 5.1; their use for estimating the percentiles of the hot-channel and average-channel coolant outlet temperatures is shown in Section 5.2; the estimation of the probability of functional failure of the system is addressed in Section 5.3.

5.1 *Building and testing the ANN and quadratic RS regression models*

RS and ANN models have been built with training sets $D_{train} = \{(\boldsymbol{x}_p, \boldsymbol{y}_p), p = 1, 2, ..., N_{train}\}$ of input/output data examples of different sizes N_{train} = 20, 30, 50, 70, 100; this has allowed extensive testing of the capability of the regression models to reproduce the outputs of the nonlinear T-H model code, based on different (small) numbers of example data. For each size N_{train} of data set, a Latin Hypercube Sample (LHS) of the 9 uncertain inputs has been drawn, $\boldsymbol{x}_p = \{x_{1,p}, x_{2,p}, …, x_{j,p}, …, x_{9,p}\}$, $p = 1, 2, …, N_{train}$. Then, the T-H model code has been run with each of the input vectors $\boldsymbol{x}_p$, $p = 1, 2, …, N_{train}$, to obtain the corresponding bidimensional output vectors $\boldsymbol{y}_p = \boldsymbol{\mu}_{\boldsymbol{y}}(\boldsymbol{x}_p) = \{y_{1,p}, y_{2,p}\}$, $p = 1, 2, …, N_{train}$ (in the present case study, the number n_o of outputs is equal to 2, i.e., the hot- and average-channel coolant outlet temperatures, as explained in Section 4.2). The training data set $D_{train} = \{(\boldsymbol{x}_p, \boldsymbol{y}_p), p = 1, 2, ..., N_{train}\}$ thereby obtained has been used to calibrate the adjustable parameters $\boldsymbol{w}^*$ of the regression models, for best fitting the T-H model code data. More specifically, the straightforward least squares method has been used to find the parameters of the quadratic RSs (Liel et al. 2009) and the common error back-propagation algorithm has been applied to *train* the ANNs (Rumelhart et al. 1986).

The choice of the ANN architecture is critical for the regression accuracy. In particular, the number of neurons in the network determines the number of adjustable parameters available to optimally fit the complicated, nonlinear T-H model code response surface by interpolation of the available training data. The number of neurons in the input layer is n_i = 9, equal to the number of uncertain input parameters; the number n_o of outputs is equal to 2, the outputs of interest; the number n_h of nodes in the hidden layer is 4 for N_{train} = 20, 30, 70 and 100, whereas it is 5 for N_{train} = 50, determined by trial-and-error.

A *validation* input/output data set $D_{val} = \{(\boldsymbol{x}_p, \boldsymbol{y}_p), p = 1, 2, ..., N_{val}\}$ made of patterns different from those of the training set D_{train} is used to monitor the accuracy of the ANN model during the training procedure: in practice, the Root Mean Square Error (RMSE) is computed on D_{val} at different phases of the training procedure. At the beginning, the RMSE computed on the validation set D_{val} typically decreases together with the RMSE computed on the training set D_{train}; then, when the ANN regression model starts overfitting the data, the RMSE calculated on the validation set D_{val} starts increasing: this is the time to stop the training algorithm. In this work, the size N_{val} of the validation set is set to 20 for all sizes N_{train} of the data set D_{train} considered, which means 20 additional runs of the T-H model code.

As measures of the ANN and RS model accuracy, the commonly adopted coefficient of determination R^2 and RMSE have been computed for each output y_l, $l = 1, 2$, on a new data set $D_{test} = \{(\boldsymbol{x}_p, \boldsymbol{y}_p), p = 1, 2, ..., N_{test}\}$ of size N_{test} = 20, purposely generated for *testing* the regression models built, and thus different from those used during training and validation.

Table 2. Coefficient of determination R^2 and RMSE associated to the estimates of the hot- and average-channel coolant outlet temperatures $T^{hot}_{out,core}$ and $T^{avg}_{out,core}$, respectively, computed on the test set D_{test} of size N_{test} = 20 by the ANN and quadratic RS models built on data sets D_{train} of different sizes N_{train} = 20, 30, 50, 70, 100; the number of adjustable parameters $\boldsymbol{w}^*$ included in the two regression models is also reported for comparison purposes.

Artificial Neural Network (ANN)							
				R^2		RMSE [°C]	
N_{train}	N_{val}	N_{test}	Number of adjustable parameters $\boldsymbol{w}^*$	$T^{hot}_{out,core}$	$T^{avg}_{out,core}$	$T^{hot}_{out,core}$	$T^{avg}_{out,core}$
20	20	20	50	0.8937	0.8956	38.5	18.8
30	20	20	50	0.9140	0.8982	34.7	18.6
50	20	20	62	0.9822	0.9779	15.8	8.7
70	20	20	50	0.9891	0.9833	12.4	6.8
100	20	20	50	0.9897	0.9866	12.0	6.3
Quadratic Response Surface (RS)							
				R^2		RMSE [°C]	
N_{train}	N_{val}	N_{test}	Number of adjustable parameters $\boldsymbol{w}^*$	$T^{hot}_{out,core}$	$T^{avg}_{out,core}$	$T^{hot}_{out,core}$	$T^{avg}_{out,core}$
20	0	20	55	0.5971	0.7914	75.0	26.6
30	0	20	55	0.8075	0.9348	51.9	14.8
50	0	20	55	0.9280	0.9353	31.7	14.6
70	0	20	55	0.9293	0.9356	31.4	14.3
100	0	20	55	0.9305	0.9496	31.2	13.1

Table 2 reports the values of the coefficient of determination R^2 and of the RMSE associated to the estimates of the hot- and average- channel coolant outlet temperatures $T^{hot}_{out,core}$ and $T^{avg}_{out,core}$, respectively, computed on the test set D_{test} by the ANN and quadratic RS models built on data sets D_{train} of different sizes N_{train} = 20, 30, 50, 70, 100; the number of adjustable parameters $\boldsymbol{w}^*$ included in the two regression models is also reported for comparison purposes.

The ANN outperforms the RS in all the cases considered. This is due to the higher flexibility in modeling complex nonlinear input/output relationships offered by the ANN with respect to the quadratic RS. Actually, if the original T-H model is not quadratic (which is often the case in practice), a second-order polynomial RS cannot be a *consistent* estimator, i.e., the quadratic RS estimates may never converge to the true values of the original T-H model outputs, even for a very large number of input/output data examples, in the limit for $N_{train} \rightarrow \infty$. On the contrary, ANNs have been demonstrated to be universal approximants of *continuous* nonlinear functions (under mild mathematical conditions) (Cybenko 1989), i.e., in principle, an ANN model *with a properly selected architecture* can be a consistent estimator of any continuous nonlinear function, e.g. any nonlinear T-H code simulating the system of interest.

5.2 *Determination of the 95th percentiles of the coolant outlet temperatures*

For illustration purposes, a configuration with N_{loops} = 3 loops is considered for the passive system of Figure 1.

The $100 \cdot \alpha^{th}$ percentiles of the hot- and average-channel coolant outlet temperatures $T^{hot}_{out,core}$ and $T^{avg}_{out,core}$ are defined as the values $T^{hot,\alpha}_{out,core}$ and $T^{avg,\alpha}_{out,core}$, respectively, such that

$$P\left(T^{hot}_{out,core} \le T^{hot,\alpha}_{out,core}\right) = \alpha \qquad (3)$$

and

$$P\left(T^{avg}_{out,core} \le T^{avg,\alpha}_{out,core}\right) = \alpha \qquad (4)$$

Figure 2 shows the Probability Density Function (PDF) of the hot-channel coolant outlet temperature $T^{hot}_{out,core}$ obtained with N_T = 250000 simulations of the original T-H model code (solid lines); the PDF of the average-channel coolant outlet temperature $T^{avg}_{out,core}$ is not shown for brevity. The same figure also shows the PDFs constructed with N_T = 250000 estimations from ANNs (dashed lines) and RSs (dot-dashed lines) built on N_{train} = 100 input/output examples.

Notice that the "true" (i.e., reference) PDF of $T^{hot}_{out,core}$ (Figure 2, solid lines) has been obtained with a very large number N_T (i.e., N_T = 250000) of simulations of the original T-H code, to provide a robust reference for the comparisons. Actually, the T-H code here employed runs fast enough to allow repetitive calculations (one code run lasts on average 3 seconds on a Pentium 4 CPU 3.00GHz): the computational time required by this reference analysis is thus $250000 \cdot 3$ s = 750000 s $\approx$ 209 h.

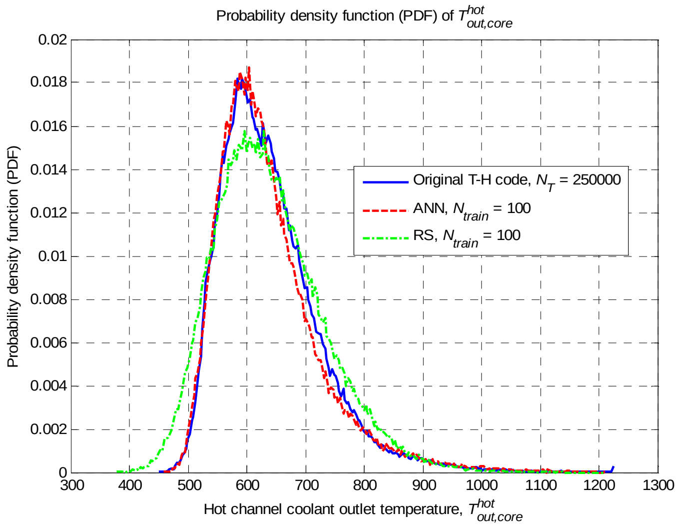

Figure 2. Hot-channel coolant outlet temperature empirical PDFs constructed with N_T = 250000 estimations from the original T-H code (solid lines) and from ANNs (dashed lines) and RSs (dot-dashed lines) built on N_{train} = 100 data examples.

The overall good match between the results from the original T-H model code and those from the ANNs and RSs regression models leads us to assert that the accuracy in the estimates can be considered satisfactory for the needs of percentile estimation in the functional failure analysis of the present T-H passive system. Also, it can be seen that the ANN estimates (dashed lines) are much closer to the reference results (solid lines) than the RS estimates (dot-dashed lines).

Table 3 reports the values of the point estimates $\hat{T}^{hot,0.95}_{out,core}$ and $\hat{T}^{hot,0.95}_{out,core}$ for the 95th percentiles $T^{hot,0.95}_{out,core}$ and $T^{avg,0.95}_{out,core}$ of the hot- and average-channel coolant outlet temperatures $T^{hot}_{out,core}$ and $T^{avg}_{out,core}$, respectively, obtained with N_T = 250000 estimations from ANNs and quadratic RSs built on N_{train} = 20, 30, 50, 70 and 100 data examples. Notice that the "true" (i.e., reference) values of the 95th percentiles (i.e., $T^{hot,0.95}_{out,core}$ = 796.31 °C and $T^{avg,0.95}_{out,core}$ = 570.22 °C) have been calculated with a very large number N_T (i.e., N_T = 250000) of simulations of the original T-H code, to provide a robust reference for the comparisons: the computational time required by the analysis is 209 h.

ANNs turn out to be quite *reliable*, providing point estimates very close to the real values in all the cases considered; on the contrary, quadratic RSs provide accurate estimates only for N_{train} = 70 and 100.

Finally, the computational times associated to the calculation of the point estimates $\hat{T}^{hot,0.95}_{out,core,BBC}$ and $\hat{T}^{hot,0.95}_{out,core,BBC}$ for $T^{hot,0.95}_{out,core}$ and $T^{avg,0.95}_{out,core}$ are compared for the two regression models: it turns out that he CPU time required by the ANNs is about 1.2 times larger than that required by the quadratic RSs, mainly due to the elaborate training algorithm needed to build the structurally complex neural model.

Table 3. ANN and quadratic RS point estimates $\hat{T}^{hot,0.95}_{out,core}$ and $\hat{T}^{avg,0.95}_{out,core}$ for the 95th percentiles $T^{hot,0.95}_{out,core}$ and $T^{avg,0.95}_{out,core}$ of the hot- and average-channel coolant outlet temperatures.

95th percentile of the coolant outlet temperatures "True" values: $T^{hot,0.95}_{out,core}$ = 796.31 °C; $T^{avg,0.95}_{out,core}$ = 570.22 °C CPU time ≈ 209 h				
Artificial Neural Networks (ANNs)				
N_{train}	N_{val}	N_{test}	$\hat{T}^{hot,0.95}_{out,core}$	$\hat{T}^{avg,0.95}_{out,core}$
20	20	20	813.50	577.36
30	20	20	810.21	575.37
50	20	20	794.95	573.64
70	20	20	795.20	571.85
100	20	20	796.70	570.84
Quadratic Response Surface (RS)				
N_{train}	N_{val}	N_{test}	$\hat{T}^{hot,0.95}_{out,core}$	$\hat{T}^{avg,0.95}_{out,core}$
20	0	20	849.98	593.15
30	0	20	827.05	583.32
50	0	20	814.48	593.89
70	0	20	806.63	573.99
100	0	20	800.81	570.32

5.3 *Functional failure probability estimation*

In this Section, ANNs and quadratic RSs are compared in the task of estimating the functional failure probability of the 600-MW GFR passive decay heat removal system of Figure 1. The previous system configuration with N_{loops} = 3 is analyzed.

Table 4 reports the values of the point estimates $\hat{P}(F)$ of the functional failure probability $P(F)$ obtained with N_T = 500000 estimations from the ANNs and quadratic RSs built on N_{train} = 20, 30, 50 ,70 and 100 data examples. Notice that the "true" (i.e., reference) value of the functional failure probability $P(F)$ (i.e., $P(F) = 3.34 \cdot 10^{-4}$) has been obtained with a very large number N_T (i.e., N_T = 500000) of simulations of the original T-H code to provide a robust term of comparison: the computational time required by this reference analysis is thus 500000•3 s = 1500000 s ≈ 417 h.

Table 4. ANN and quadratic RS point estimates $\hat{P}(F)$ for the functional failure probability $P(F)$.

Functional failure probability ("True" value: $P(F) = 3.34 \cdot 10^{-4}$; CPU time ≈ 417 h)			
Artificial Neural Networks (ANNs)			
N_{train}	N_{val}	N_{test}	$\hat{P}(F)$
20	20	20	$1.01 \cdot 10^{-4}$
30	20	20	$1.53 \cdot 10^{-4}$
50	20	20	$2.45 \cdot 10^{-4}$
70	20	20	$3.01 \cdot 10^{-4}$
100	20	20	$3.59 \cdot 10^{-4}$
Quadratic Response Surface (RS)			
N_{train}	N_{val}	N_{test}	$\hat{P}(F)$
20	0	20	$9.81 \cdot 10^{-5}$
30	0	20	$1.00 \cdot 10^{-4}$
50	0	20	$2.15 \cdot 10^{-4}$
70	0	20	$2.39 \cdot 10^{-4}$
100	0	20	$3.17 \cdot 10^{-4}$

It can be seen that as the size of the training sample N_{train} increases, both the ANN and quadratic RS provide increasingly accurate estimates of the true functional failure probability $P(F)$, as one would expect. On the other hand, in the cases of small training sets (e.g., N_{train} = 20, 30 and 50) the functional failure probabilities are significantly underestimated by both the ANN and the quadratic RS models (e.g., the point estimates $\hat{P}(F)$ for $P(F)$ lie between $9.81 \cdot 10^{-5}$ and $2.45 \cdot 10^{-4}$). However, in these cases of small data sets available the analyst would still be able to correctly estimate the order of magnitude of a *small* failure probability (i.e., $P(F) \sim 10^{-4}$), in spite of the *low* number of runs of the T-H code performed to generate the N_{train} = 20, 30 or 50 input/output examples.

Finally, it is worth noting that although ANNs provide better estimates than quadratic RSs, the difference in the performances of the two regression models is less evident than in the case of percentile estimation (Section 5.2). This may be due to the fact that estimating the value of the functional failure probability $P(F)$ is a simpler task than estimating the exact values of the corresponding coolant outlet temperatures.

6 CONCUSIONS

In this paper, ANNs and quadratic RSs have been compared in the task of estimating, in a fast and efficient way, the probability of functional failure of a T-H passive system. A case study involving the natural convection cooling in a GFR after a LOCA has been taken as reference.

ANN and quadratic RS models have been constructed on the basis of sets of data of limited, varying sizes, which represent examples of the nonlinear relationships between 9 uncertain inputs and 2 relevant outputs of the T-H model code (i.e., the hot- and average-channel coolant outlet temperatures). Once built, such models have been used, in place of the original T-H model code, to: compute the temperatures 95th percentiles of the hot-channel and average-channel temperatures of the coolant gas leaving the reactor core; estimate the functional failure probability of the system by comparison of the computed values with predefined failure thresholds. In all the cases considered, the results have demonstrated that ANNs outperform quadratic RSs in terms of estimation accuracy: as expected, the difference in the performances of the two regression models is more evident in the estimation of the 95th percentiles than in the (easier) task of estimating the functional failure probability of the system. Due to their flexibility in nonlinear modeling, ANNs have been shown to provide more reliable estimates than quadratic RSs even when they are trained with very low numbers of data examples (e.g., 20, 30 or 50) from the original T-H model code.

On the basis of the results obtained, ANNs can be considered more effective than quadratic RSs in the estimation of the functional failure probability of T-H passive systems because they provide more *accurate* (i.e., closer to the true values) estimates; on the other hand, the computational time required by ANNs is somewhat longer than that required by quadratic RSs, due to the elaborate training algorithm for building the structurally complex neural model.

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